Optimality Criterion Techniques Applied to Frames Having General Cross-Sectional Relationships

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An optimality criterion-based method developed in this research is capable of locating the optimal design of WF steel frames with members having general cross-sectional relationships among the area, moment of inertia, and section modulus. The method uses the idea of one most critical constraint to avoid the calculation of large sets of Lagrange multipliers, and also avoids the scaling procedure normally used in other optimality criterion techniques. Several example structures, designed under stress, displacement, and minimum area constraints, demonstrate the efficiency, reliability, and simplicity of the method. Although the method of this research uses the idea of one most critical constraint, the optimal designs presented in this paper have several active constraints.

Introduction

OPTIMALITY criterion techniques have been applied to many structural design problems with a great potential during the past few years (see Ref. 1 for an excellent review article on this subject). Recently, Khan et al.[2] have developed several very efficient techniques based on optimality criteria for the design of trusses, frames, and other complex structures, such as wingbox structures. Also, problems involving more than one design variable per member have been considered.[2-5] The techniques, based on the assumption that only one constraint was most critical at any stage of the process, have proven to be very efficient in terms of computer time and core storage. The optimal designs obtained by these new methods have compared favorably to those obtained by previous methods.

In this paper, a new procedure is developed for designing WF steel frames with members having complex nonlinear cross-sectional relationships among the area, moment of inertia, and section modulus. In 1966, Brown and Ang[6] proposed a cross-sectional relationship among the area, moment of inertia, and section modulus to reflect the AISC code for WF steel sections. In 1977, Calafell and Willmert[7] modified the cross-sectional fits proposed by Brown and Ang to better represent commercially available economy WF members. These modified cross-sectional relationships have been used in this research. The design procedure of this research avoids the scaling of the design variables normally used in other optimality criterion mehtods. Elimination of the scaling procedure was essential due to the fact that the structural stiffness matrix and the corresponding displacement vector cannot be scaled exactly when the area and moment of inertia are nonlinearly related.

The research presented herein locates the optimal designs of framed structures, under stress, displacement, and minimum area constraints efficiently.

Theory

The structural design problem considered here can be stated as: Find the vector of design variables A_i, $i=1,...,N$ such that the total volume of the structure

$$V=\sum_{i-1}^{N} A_i L_i \quad \text{is minimum} \tag{1}$$

subjected to constraints

$$\sigma_i \leq \bar{\sigma}_i \qquad i=1,...,N \tag{2}$$

$$u_j \leq \bar{u}_j \qquad j=1,...,J \tag{3}$$

where A_i and L_i are the cross-sectional area and length of the ith member, N the number of members, σ_i and $\bar{\sigma}_i$ the stress and its limiting value in the ith member, u_j and $\bar{u}_j$ the jth constrained displacement and its limiting value, and J the number of constrained degrees of freedom. The stress σ_i includes both axial and bending stress.

Stress Constraints

For stress-constrained problems, stated by Eqs. (1) and (2) only, the well-known stress ratio recursion formula (see, for example, Ref. 3) is used.

$$(A_i)_{v+1} = \left[\left(\frac{\max|\sigma_i|}{\bar{\sigma}_i}\right)A_i\right]_v \tag{4}$$

where v is the iteration counter. However, to obtain a more effective algorithm, Eq. (4) is modified somewhat as follows:

$$(A_i)_{v+1} = \left[\left(\frac{\max|\sigma_i|}{C_l\bar{\sigma}_i}\right)A_i\right]_v, \qquad \text{if } \sigma_i > \bar{\sigma}_i \tag{5}$$

$$(A_i)_{v+1} = \left[\left(\frac{\max|\sigma_i|}{\bar{\sigma}_i}\right)^{\beta}A_i\right]_v, \qquad \text{if } \sigma_i \leq \bar{\sigma}_i \tag{6}$$

where C_l is a constant and ranges between 0.990 and 0.999. When recursion equation (4) is used, the stress σ_i approaches $\bar{\sigma}_i$ as the design comes close to the optimal value. However, when the optimization is nearly converged, in some problems certain members are a little overstressed.

This difficulty can be easily overcome by scaling the design variables. But scaling is not possible in the problems considered in this paper. Therefore, Eq. (5) was used instead. The term $C_l\bar{\sigma}_i$ represents the limiting value of stress rather than

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$\bar{\sigma}_i$. As the design process continues, all of the members whose stresses are greater than $\bar{\sigma}_i$ at any iteration are forced to a value of $C_1\bar{\sigma}_i$. Equation (6) is the recursion relation for redesign of those members that have σ_i less than $\bar{\sigma}_i$. Exponent β (ranging from 0.30 to 1.0) changes the values of the design variables more slowly (similar to the move limits used in mathematical programming method) thereby avoiding the oscillations in the design algorithm.

Experience indicates that the combined use of Eqs. (5) and (6) is much more reliable and stable than Eq. (4).

The use of Eq. (4) to iterate an area for stress constraints is not unique. References 4, 5, 11, and 12 describe the use of moment of inertia, thickness, or section height as the design variable. For plate and truss (rod) elements, the thickness and cross-sectional area are the most appropriate design variables. However, for indeterminate frame structures where the axial and bending effects are included, the area, section modulus, or moment of inertia can be used as the design variable. As explained in Ref. 3, it is simple and convenient to use area as the design variable. As explained earlier, in this research, the use of area as the design variable is extremely reliable.

Displacement Constraints

Considering the displacement constraints alone, the Lagrangian for the design problem of set of Eqs. (1) and (3) is

$$L = V + \sum_{j=1}^{J} \lambda_j (u_j - \bar{u}_j)$$

and the Kuhn-Tucker conditions for a minimum are

$$\frac{\partial V}{\partial A_i} + \sum_{j=1}^{J} \lambda_j \frac{\partial u_j}{\partial A_i} = 0 \qquad i = 1,\ldots,N$$

$$u_j - \bar{u}_j \leq 0 \text{ or } \lambda_j \geq 0 \qquad j = 1,\ldots,J \tag{7}$$

Let us suppose that the pth constrained displacement is most critical (active), and the other constrained displacements are not. Then Eqs. (7) become

$$\frac{\partial V}{\partial A_i} + \lambda_p \frac{\partial u_p}{\partial A_i} = 0 \qquad i = 1,\ldots,N$$

$$u_p - \bar{u}_p = 0 \qquad \lambda_{pq} > 0 \tag{8}$$

By means of the unit load theorem of structural analysis, the derivative in the first set of Eqs. (7) is given by

$$\frac{\partial u_p}{\partial A_i} = -X_i^T \frac{\partial K_i}{\partial A_i} \bar{X}_i = -g_i \tag{9}$$

where K_i is the stiffness matrix of the ith membr, X_i and $\bar{X}_i$ are the displacement vectors for the ith member due to the actual loads and a unit load applied at the location and in the direction of the pth constrained degree of freedom, respectively. (Note: $\partial K_i/\partial A_i$ can easily be computed in the closed form. See Ref. 8 for details.) At the optimal design all those members in the set of Eqs. (8) that have $\partial u_i/\partial A_i < 0$ will be satisfied for a single value of the Lagrange multiplier λ_p. For nonoptimal designs it is desired to obtain a value of λ_p that most closely satisfies these equations. Defining a residual R_k, using the set of Eqs. (8) and (9), as

$$R_k = L_k - \lambda_p g_k \tag{10}$$

where λ_p will be taken as the value that minimizes the sum of the squares of residuals,[9] i.e., the value for which

$$\frac{d}{d\lambda_p}\left[\sum_{k=1}^{N_1} R_k^2\right] = 0 \tag{11}$$

where N_1 includes only those members which have positive value of g_i.

Equation (11) results in the following value for λ_p.

$$\lambda_p = \sum_{k=1}^{N_1} L_k g_k \Big/ \sum_{k=1}^{N_1} g_k^2 \tag{12}$$

Substituting this into the set of Eqs. (8), the Lagrange multiplier is eliminated from the optimality conditions

$$1 = \left(\sum_{k=1}^{N_1} L_k g_k \Big/ \sum_{k=1}^{N_1} g_k^2\right) \frac{g_i}{L_i} \qquad i = 1,\ldots,N$$

$$1 = u_p / \bar{u}_p \tag{13}$$

First multiply both sides of the set of Eqs. (13) and then multiply by A_i the optimality conditions given by the set of Eqs. (13) can be combined to form only one recursion equation as

$$(A_i)_{v+1} = \left[\left(\frac{u_p}{\bar{u}_p}\right)\left\{\left(\sum_{k=1}^{N_1} L_k g_k \Big/ \sum_{k=1}^{N_1} g_k^2\right)\frac{g_i}{L_i}\right\} A_i\right]_v \tag{14}$$

Ideally, Eq. (14) could be used to change the values of the design variables from one iteration to the next. But this simple linear form of Eq. (14) does not work in all cases as a recursion equation unless, like some mathematical programming methods, move limits are imposed on the design variables. For faster and more stable rates of convergence to the optimal design, a modified recursion equation was developed as follows:

$$(A_i)_{v+1} = \left[\left(\frac{u_p}{C_2 \bar{u}_p}\right)^{\alpha}\left\{\left(\sum_{k=1}^{N_1} L_k g_k \Big/ \sum_{k=1}^{N_1} g_k^2\right)\frac{g_i}{L_i}\right\}^{\eta} A_i\right]_v \tag{15}$$

where α is an exponent that ranges between 0.65 and 1.0. However, for most of the sample problems considered in this research, $\alpha = 0.75$ gave the best results. C_2 is a constant that ranges between 0.985 to 0.999 when $u_p > \bar{u}_p$; otherwise its value is 1. The relaxation parameter η ranges between 0.003 and 0.6.

The values of α and η less than 1 slow down the fast changes in the design variables and thus stabilize the design process. At the optimal design, the optimality conditions, the set of Eqs. (13), will be satisfied; therefore, the values of α and η will not make any difference to the optimal design obtained, but only control the rate at which this design is obtained.

The recursion equations [(5), (6), and (15)], have been incorporated into the following design algorithm, which is applicable to stress- and displacement- constrained frame structures.

Design Algorithm

1) Choose any uniform design A_i, $i = 1,2,\ldots,N$.

2) Analyze the design.

3) Check all the displacement constrained degrees of freedom and determine the node and direction for which the calculated displacement most closely approaches (or exceeds) the allowable displacement. This is the most critical

displacement, u_p. Calculate

$$\Delta_{CR} = |u_p| / |\bar{u}_p|$$

the most critical displacement response ratio.

4) Compute the maximum stress max $|\sigma_i|$ in each member i. Also, determine the value R_i for each element as

$$R_i = 1 - \frac{\max |\sigma_i|}{|\bar{\sigma}_i|}$$

5) Apply a unit load only at the node and in the direction of the most active displacement constraint. Let the set of resulting nodal displacements be denoted by $\bar{X}$.

6) Compute the displacement derivatives

$$g_i = \frac{\partial u_p}{\partial A_i} = -X_i^T \frac{\partial K_i}{\partial A_i} \bar{X}_i$$

and the Lagrange multiplier associated with the critical displacement

$$\lambda_p = \sum_{j=1}^{J} g_j L_j \bigg/ \sum_{j=1}^{J} g_j^2$$

where summation J holds only for those members for which $g_j > 0$.

7) Group the members as follows:

i) If $\sigma_i < \bar{\sigma}_i$ and $0 \leq R_i \leq E_1$, member i belongs to group G_1, where E_1 ranges from 0.01 to 0.005.

ii) If $g_i < 0$ and $\sigma_i \leq \bar{\sigma}_i$, member i belongs to group G_2.

iii) If $\sigma_i > \bar{\sigma}_i$, member i belongs to group G_3.

iv) If $\sigma_i < \bar{\sigma}_i$ for all member and $\Delta_{CR} > 1$, every member belongs to group G_4. (Note: either all members would be in this group or none.)

v) Otherwise, member i belongs to group G_5. Note that any particular group could be empty and a particular member would belong to only one group at a time.

8) Do not resize the members of group G_1.

9) Use the recursion equation

$$(A_i)_{v+1} = \left[\left(\frac{\max |\sigma_i|}{\bar{\sigma}_i} \right)^{0.55} A_i \right]_v$$

to resize the members of group G_2.

10) Use the recursion equation

$$(A_i)_{v+1} = \left[\left(\frac{\max |\sigma_i|}{0.995 \bar{\sigma}_i} \right) A_i \right]_v$$

to resize the members of group G_3.

11) Resize the members of group G_4 as

$$(A_i)_{v+1} = \left[\left(\frac{u_p}{0.997 \bar{u}_p} \right)^{0.75} \left(\lambda_p \frac{X_i^T (\partial K / \partial A_i) \bar{X}_i}{L_i} \right)^{\eta} A_i \right]_v$$

12) Resize the members of group G_5 as

$$(A_i)_{v+1} = \left[\left(\frac{u_p}{\bar{u}_p} \right)^{0.75} \left(\lambda_p \frac{X_i^T (\partial K / \partial A_i) \bar{X}_i}{L_i} \right)^{\eta} A_i \right]_v$$

13) Calculate the ratio

$$\Delta V = \frac{|V_0 - V_{v+1}|}{V_{v+1}}$$

and if ΔV is less than a prescribed value (say, 0.00001 to 0.005) then stop; otherwise go to step 14. The converged design of this step is the optimal design, where V_0 is the smallest volume obtained among all previous iterations satisfying all constraints and V_{v+1} is the volume obtained from the current feasible design.

14) If after iteration 4 the design is oscillating (see section on Explanation of the Design Algorithm), then change the value of η to $C_3 \eta$, where C_3 is a constant which ranges from 0.2 to 0.9. If the oscillating designs have all constraints satisfied, a small value of C_3 (i.e., 0.3 or 0.4) is recommended; otherwise a large value of C_3 (i.e., 0.8, 0.85, or 0.9) is chosen. (For details see section on Explanation of the Design Algorithm.) With the new value of η, go to step 2 and continue the design process.

Explanation of the Design Algorithm

General

In the design algorithm above, an initial design (usually far from the optimal design) and starting value of α (range between 0.7 and 0.80) and η (range 0.3 to 0.35) are selected. After a structural analysis is performed, step 7 of the procedure assigns the members to one of five groups. Steps 8 to 12 are used to resize the members of different groups. If the design of the current iteration has all constraints satisfied, then this is a feasible design (not necessarily an optimal design) and is denoted by V_0. As the iteration process continues, if another feasible design is obtained, it is then compared with the previous satisfactory design V_0. If the volume of the two feasible designs is within a prescribed tolerance ΔV (within the range 0.00001 to 0.005) then the least weight design between the two is selected as the optimal design. It is possible that, when the prescribed tolerances are too small, the procedure could get into an almost infinite loop because of roundoff errors. It might be of value to stop the procedure after a prescribed number of iterations. When this happens the best feasible design obtained among all iterations could be selected as the optimal design. However, in all of the example problems considered in this research, the technique converged without this additional stop criterion.

The convergence and stability of the algorithm are controlled by choosing the starting values of α and η from the given ranges. In order to further stabilize the process, the value of η is reduced when oscillations occur. [This is checked if $(V_{v-2} - V_{v-1})(V_{v-i} - V_v) < 0$.] When the oscillating designs have all constraints satisfied, the η is reduced to 0.3 times the original η (or 0.4η). This is due to the fact that small changes could be made in the design variables in further iterations to come, and by doing this the optimal design is converged in a few iterations without continuous oscillations in the design process. If the oscillating designs are not feasible

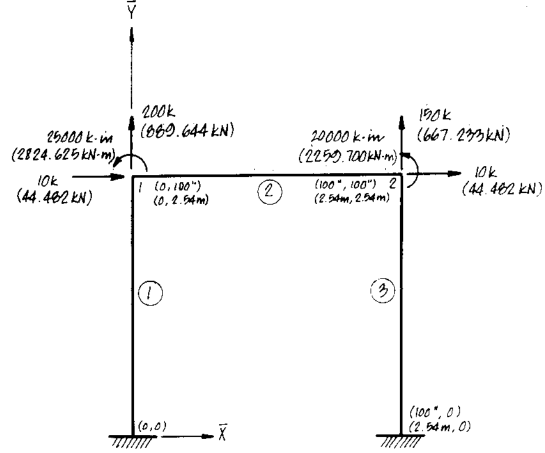

Fig. 1 Three-member frame.

designs, then η is reduced to 0.75η (or 0.8η). This way η is reduced gradually and the optimal design is obtained by avoiding large oscillations. To reduce the value of η at a greater rate reduces the total number of iterations to converge to the optimal design. However, η could be reduced at a faster rate when the optimal design is completely displacement-dominated, i.e., all stress constraints are inactive. The lowest value of η that can be achieved is 0.003. If η reaches its lowest value in any design process, no further reduction in η is made. The use of the exponent η is similar to move limits used in mathematical programming methods. In mathematical programming techniques, move limits put a restriction on the maximum change in the design variables from one iteration to the next, thereby helping to stabilize the design process.

Resizing Groups

Group G_1

Members of this group are at their optimal values. Any further reduction or change in their values essentially results in more iterations. This is a very effective group and does not involve any computational efforts.

Group G_2

Members of this group are designed using the stess ratio formula. However, the ratio has an exponent value of 0.55 (ranging between 0.4 and 1.0) that changes the values of A_i slowly, thus avoiding oscillations.

Group G_3

Members of this group are overstressed. The stress ratio recursion formula changes the design variables. The stress ratio is divided by a factor normally equal to 0.99, but could range between 0.98 and 0.999. This factor helps to bring the design within (but very close to) the feasible boundaries of the stress constraints. When this factor is 1 the stress constraints can be satisfied within a prescribed tolerance. However, at the optimal design, the stress in the members may be a little over their prescribed values. When this factor is 0.99 (or less than 1), all of the stress constraints at the optimal design are satisfied.

Group G_4

Members of this group are completely displacement-dominated, i.e., all stress constraints are satisfied. The exponent 0.75 slowly brings the most critical displacement, u_p, up to its limiting value $\bar{u}_p$. There is also a factor 0.995 that brings u_p within (but close to) the feasible boundary of displacement constraint. This factor is used for the same reasons as explained above in resizing the members of group G_3.

Group G_5

Members of this group are designed when the displacement derivatives are negative and the member stresses are less than their maximum specified values. No factor (i.e., 0.99) is used in the recursive equation to design members of this group. This is because at the optimal u_p converges to $\bar{u}_p$ and not at $0.99\bar{u}_p$; hence, a heavier design is not produced.

Examples

To show the effectiveness and efficiency of the design algorithm, several frames were designed. Some of them are discussed in this section. All of the frames presented here have some common data; that is, each member of the frame is treated as one finite element. Axial shear and bending moment are included in the formulation, resulting in six degrees of freedom per joint. The material is steel with $E=206{,}842{,}710$ MPa and $p=7{,}833.412$ kg/m^3. The stress limit for all members is 165.474 MPa. One load condition is considered.

The following relationships proposed in Ref. 7 were used among area A, section modulus S, and moment of inertia I.

$$
\begin{aligned}
S &= 1.6634A^{1.511}, && 0 \le A \le 15 \\
&= [281.077A^2 + 84100]^{1/2} - 290, && 15 < A \le 44 \\
&= 13.761A - 103.906, && 44 < A \le 100 \\
I &= 4.592A^2, && 0 \le A \le 15 \\
&= 4.638A^2, && 15 < A \le 44 \\
&= 256.229A - 2300, && 44 < A \le 100
\end{aligned}
$$

where A is the area in square inches.

Three-Member Frame

The structure and the loading is shown in Fig. 1. The horizontal displacements of all joints were limited to ± 0.00127 m and the stress limits above were used for all members. Table 1 shows the final designs and compares them with the results obtained by the SUMT (Fiacco and McCormick penalty function along with the Powell's search scheme) technique and the CONMIN code.[10] The final design has the horizontal displacements of joints 1 and 2 active. Thus there are two active displacement constraints at the optimal. Also, members 1 and 2 have stress equal to their limiting value. The optimality criterion method of this research chooses the horizontal displacement of joint 1 as the most

Table 1 Final designs of the three-member frame

Technique		SUMT		CONMIN		This paper
n		—		—		0.35
a		—		—		0.75
$\bar{u}_{max}$, m		0.00127		0.00127		0.00127
Analyses		1202		43		20
Starting areas, m^2		0.025806		0.012903		0.064516
Member No.	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa
1	0.011343	165.474	0.011359	165.336	0.011347	165.405
2	0.050813	165.474	0.050806	165.474	0.050811	165.474
3	0.034581	109.696	0.034444	109.213	0.034578	109.696
Volume, m^3		0.2457		0.2462		0.2457
CPU time, s		80		8		1.76
d_1, m		−0.001270		−0.001267		−0.001270
d_4, m		−0.001262		−0.001259		−0.001262

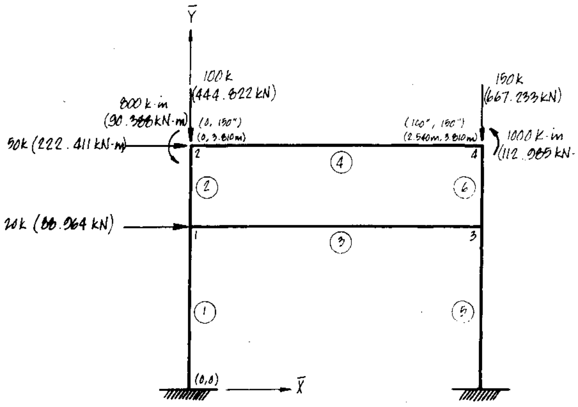

Fig. 2 Six-member frame.

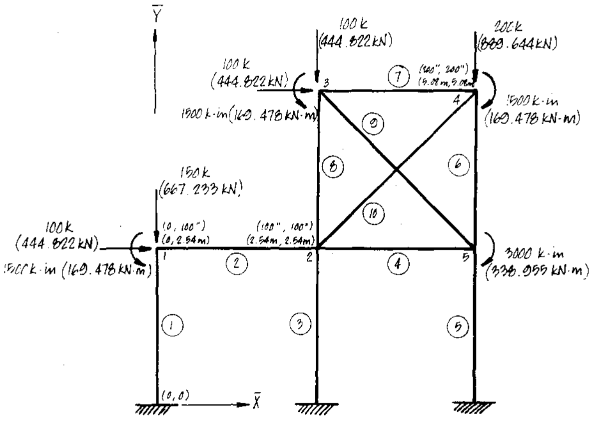

Fig. 3 Ten-member frame.

critical displacement constraint during the design process and gives an optimal design that has multiple active constraints. The initial design for this research, the SUMT, and the CONMIN, were 0.064516, 0.025806, and 0.012903 m^2, respectively.

Six-Member Frame

This structure is loaded as shown in Fig. 2. The horizontal displacements of all joints were limited to 0.00254 m. The stress limits above were used for all members. A minimum area limit of 0.00129 m^2 was used. Table 2 shows the final designs obtained by this research, the SUMT, and the CONMIN code. The optimality criterion method and the SUMT gave very close optimal designs. The CONMIN code produced an approximate optimal design and took 167 analyses. At the optimal design the horizontal displacement of joint 2 was active. Also, members 1 and 6 have stress equal to their limiting value. The starting design for this research, the SUMT, and the CONMIN, were 0.064516, 0.025806, and 0.012903 m^2, respectively.

Ten-Member Frame

This structure and the loading are shown in Fig. 3. The horizontal displacements of all joints were limited to 0.00254 m. The stress limits above were used for all members. A minimum area limit of 0.003226 m^2 was used. Table 3 shows the results obtained for this structure. This table indicates that the optimal design obtained by this research is in excellent agreement with those obtained by the SUMT. However, the final design obtained by the CONMIN is 17% heavier. Several uniform starting design and the default parameters were used, but no other lightweight design was obtained by the CONMIN code. The optimal design of this structure has the horizontal displacement of nodes 3 and 4 active. The starting design for this research and the SUMT were uniform 0.064516 and 0.025806 m^2, respectively.

Fifteen-Member Frame

This structure is loaded as shown in Fig. 4. The horizontal displacements of nodes 1, 2, 3, 7, 8, and 9 were limited to 0.00762 m, and the stress limits above were used for all members. A minimum area limit of 0.003226 m^2 was used. Table 4 shows the results obtained by this research and compares them with those obtained by the SUMT method. The total volume obtained by both methods was almost the same, but designs were slightly different, indicating a flat optimum. The total number of analyses and CPU time taken by this research is substantially less than those used by the SUMT. At the optimal the horizontal displacement of nodes 3 and 9 were active. The stresses in members 14 and 15 were close to their limiting value. The starting design for this

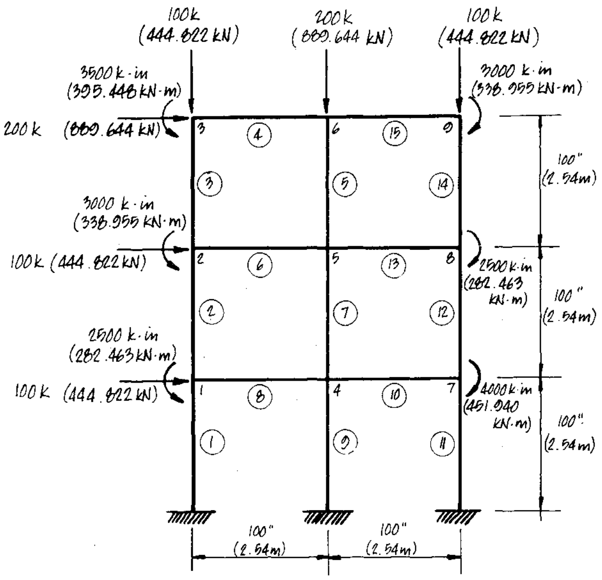

Fig. 4 Fifteen-member frame.

research and the SUMT were uniform 0.064516 and 0.038710 m^2, respectively.

Twenty-Five-Member Frame

The structure and the loading are shown in Fig. 5. The horizontal displacements of nodes 1, 2, 3, 10, 11, and 12 were limited to 0.00127 m, and the stress limits above were used for all members. A minimum area limit of 0.003226 m^2 was used. Table 5 shows the results obtained by this research and compares them favorably with those obtained by the SUMT method. The total volume obtained by the SUMT is slightly higher than that obtained by this research. The total number of analyses and CPU time taken by this research are drastically less than those used by the SUMT. At the optimal, the horizontal displacement of nodes 2 and 11 were active. The stresses in nine members were close to their limiting value. The starting design for this research and the SUMT were uniform, 0.0064516 and 0.054839 m^2, respectively.

Seventy-Member Frame

The structure and the loading are shown in Fig. 6. The horizontal displacements of all joints are limited to 0.03810 m and the stress limits above were imposed for all members. The minimum area limit of 0.003226 m^2 was used. The final design obtained is given in Table 6. At the optimal the horizontal displacement of the joint under the horizontal load

Table 2 Final designs of the six-member frame

Technique		SUMT		CONMIN		This paper
n		—		—		0.35
a		—		—		0.75
$\bar{u}_{max}$, m		0.00254		0.00254		0.00254
Analyses		2101		167		19
Starting areas, m^2		0.025806		0.012903		0.064516
Member No.	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa
1	0.003092	165.474	0.003099	165.198	0.003103	165.474
2	0.008734	136.681	0.008146	148.099	0.008720	137.206
3	0.012268	120.107	0.013167	112.247	0.012790	115.763
4	0.001291	56.813	0.001290	59.915	0.001290	58.468
5	0.020057	136.034	0.019734	137.137	0.019541	139.205
6	0.008357	165.474	0.008353	165.474	0.008355	165.405
Volume, m^3		0.1150		0.1157		0.1150
CPU time, s		88.86		16		2.19
d_4, m		0.002540		0.002290		0.002540

Table 3 Final designs of the ten-member frame

Technique		SUMT		CONMIN		This paper
n		—		—		0.40
a		—		—		0.80
$\bar{u}_{max}$, m		0.00254		0.00254		0.00254
Analyses		4012		69		31
Starting areas, m^2		0.025806		0.012903		0.064516
Member No.	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa
1	0.028243	92.597	0.025270	98.043	0.027547	91.631
2	0.020405	57.364	0.020935	63.432	0.021418	55.710
3	0.020905	80.462	0.022716	74.601	0.022066	72.533
4	0.003226	57.778	0.013632	64.673	0.003226	60.605
5	0.042470	85.909	0.029832	104.594	0.042286	85.840
6	0.017770	109.213	0.014666	101.008	0.017401	110.799
7	0.003226	81.220	0.012441	27.855	0.003226	78.187
8	0.003226	145.755	0.014357	55.020	0.010776	140.033
9	0.010346	149.547	0.018793	52.400	0.010776	142.859
10	0.011610	26.130	0.013557	14.203	0.010537	27.717
Volume, m^3		0.4330		0.5068		0.4330
CPU time, s		237.56		15		6.91
d_7, m		0.002518		0.002484		0.002503
d_{10}, m		0.002540		0.002534		0.002539

of 444.822 kN was exactly active. The maximum stress in members 1 and 69 were 151.202 and 154.167 MPa, respectively. The starting design for this example was 0.064516 m^2. The optimal design is completely displacement-dominated.

Discussion of Results

In this paper, six examples were considered, ranging from a small structure to a large structure. Three-, six-, ten-, fifteen-, and twenty-five-member frames were designed by the SUMT and the optimality criterion method. It is interesting to note that both of the methods gave almost the same optimal designs. The optimal designs have several active stress and displacement constraints, but the optimality criterion method of this research uses the idea of one most critical (violated) displacement constraint during the design process. The optimality criterion method took a considerably lower number of analyses as compared to the SUMT method for all examples considered. Also, the starting designs for this research were taken much further from the optimal designs compared to the SUMT method. The optimality criterion method took 10-50 analyses regardless of the problem size.

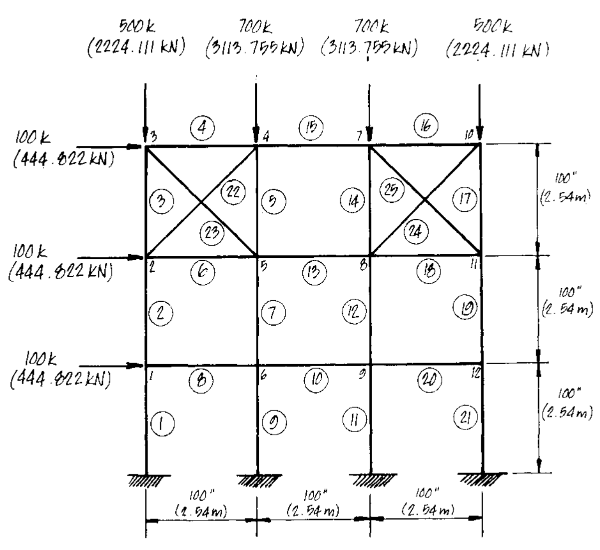

Fig. 5 Twenty-five-member frame.

Table 4 Final designs of the fifteen-member frame

Technique		SUMT		This paper
n		—		0.3
a		—		0.75
$\bar{u}_{max}$, m		0.00762		0.00762
Analyses		6258		28
Starting areas, m^2		0.038710		0.064516
Member No.	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa
1	0.028038	131.000	0.028379	136.999
2	0.013947	110.868	0.016623	103.077
3	0.025523	97.492	0.026043	94.872
4	0.020784	77.428	0.021003	77.152
5	0.028387	100.319	0.028302	103.559
6	0.028387	101.767	0.028169	98.319
7	0.028387	140.102	0.028482	131.414
8	0.019314	101.008	0.022642	99.078
9	0.028385	118.038	0.028248	126.381
10	0.034345	132.724	0.031398	137.413
11	0.047117	134.861	0.043952	137.068
12	0.028387	112.729	0.028373	110.799
13	0.028677	115.556	0.028567	113.488
14	0.005220	165.474	0.005771	165.336
15	0.011636	144.928	0.010849	160.165
Volume, m^3		0.9572		0.9571
CPU time, s		787		7.80
d_7, m		0.007620		0.007162
d_{25}, m		0.007404		0.007405

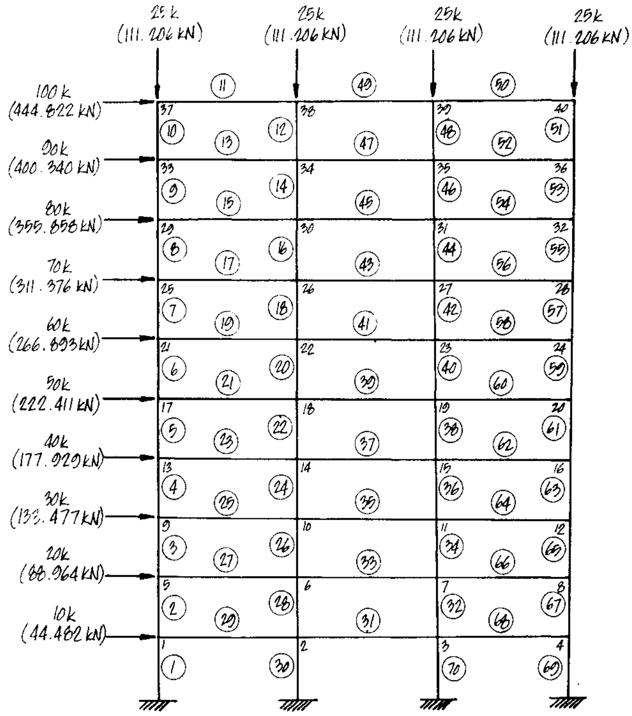

Fig. 6 Seventy-member frame.

Table 5 Final designs of the twenty-five-member frame

Technique		SUMT		This paper
n		—		0.32
a		—		0.73
$\bar{u}_{max}$, m		0.00127		0.00127
Analyses		10016		34
Starting areas, m^2		0.054839		0.064516
Member No.	Area, m^2	Stress, MPa	Area, m^2	Stress, MPa
1	0.012685	165.474	0.017445	165.336
2	0.009036	165.474	0.012514	165.474
3	0.006146	165.474	0.010600	165.405
4	0.011530	111.419	0.003877	68.189
5	0.018670	165.474	0.016717	165.061
6	0.003229	59.157	0.003226	24.407
7	0.041611	165.474	0.025444	165.474
8	0.019900	53.434	0.024121	51.780
9	0.032697	165.405	0.029032	165.061
10	0.036056	79.772	0.017036	40.128
11	0.034599	147.203	0.019408	164.716
12	0.022581	165.474	0.033329	135.413
13	0.005833	60.260	0.026626	45.643
14	0.019488	165.474	0.017107	164.716
15	0.003226	53.848	0.005182	71.085
16	0.003227	44.609	0.003226	54.607
17	0.013491	165.474	0.013575	165.405
18	0.016206	53.641	0.023638	57.227
19	0.026210	124.519	0.037937	112.591
20	0.028387	67.017	0.044693	81.082
21	0.048023	96.802	0.064516	100.664
22	0.015093	75.360	0.011060	83.633
23	0.042354	100.526	0.014929	93.976
24	0.003230	70.189	0.003226	75.291
25	0.004562	108.179	0.009836	120.176
Member No.		782,922		78,194
CPU time, s		2695		14.35
d_4, m		0.001270		0.001269
d_{31}, m		0.001199		0.001258

Table 6 Final designs of the seventy-member frame

Technique		This paper	
n	0.38		
a	0.75	Volume, m^3	5.580
Analyses	20	CPU time, s	140
Member No.	Area, m^2	Member No.	Area, m^2
1	0.064516	36	0.029959
2	0.064516	37	0.029213
3	0.064299	38	0.029067
4	0.061915	39	0.028734
5	0.049181	40	0.028890
6	0.037350	41	0.028382
7	0.028260	42	0.028661
8	0.018897	43	0.028217
9	0.011706	44	0.028517
10	0.016538	45	0.028230
11	0.021359	46	0.028967
12	0.027640	47	0.028903
13	0.021128	48	0.015436
14	0.024089	49	0.014232
15	0.017854	50	0.003226
16	0.026588	51	0.003226
17	0.025952	52	0.015452
18	0.028269	53	0.014961
19	0.028485	54	0.028420
20	0.028603	55	0.025955
21	0.028969	56	0.028746
22	0.029704	57	0.031274
23	0.029606	58	0.028535
24	0.030295	59	0.041964
25	0.029910	60	0.029102
26	0.030915	61	0.053725
27	0.031052	62	0.028975
28	0.030076	63	0.064121
29	0.029160	64	0.029095
30	0.029651	65	0.064361
31	0.026975	66	0.030111
32	0.030526	67	0.064516
33	0.030041	68	0.028694
34	0.031501	69	0.064516
35	0.030461	70	0.028428

An attempt was also made to design a few frame examples by the CONMIN code. This code produced approximate (but close) optimal designs for the 3- and 6-member frames, but the optimal design for the 10-member frame was 17% heavier as compared to the SUMT or optimality criterion method. The use of the CONMIN code was not too successful. It is felt that further experimentation is needed to make the best use of this code.

All of the examples presented in this paper were designed on the IBM 4341 computer.

Conclusion

A new design procedure, based on an optimality criterion technique, has been developed for stress- and displacement-constrained frames with complex nonlinear cross-sectional fits. To check the accuracy, efficiency, and reliability of this procedure, several frame examples were designed, and results were compared with a SUMT technique (Fiacco and McCormick penalty function along with the Powell's search scheme). In all cases the results obtained by this research matched exactly with those obtained by the SUMT method. The optimality criterion method of this research requires a fraction of CPU time as compared to the SUMT. The total core storage required for the program is slightly more than that used for structural analysis alone. The results indicate that the method of research is highly efficient, simple, and reliable.

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